Centrality: Who Are the Most Important Actors?

- Measures of *importance* or *prominence* all attempt to describe a property of actor location in a network.
  - ✓ Important actors typically occupy strategic location in a network.
- Definitions of importance include:
  - ✓ *degree*
  - ✓ *closeness*
  - ✓ *betweenness*
  - ✓ *information*
  - ✓ *differential status* or *rank*
- These measures are typically calculated for individuals, but may also be aggregated to graphs or subgraphs.
- Most measures of *centrality* are used in conjunction with symmetric sociomatrices and measure dichotomous relations.
- Measures of *prestige*, on the other hand, require directed graphs.

Prominence: Centrality and Prestige

- Assumptions:
  - ✓ Measurements on a single, dichotomous relation.
- An actor is *prominent* if the ties of the actor make the actor particularly visible to other actors in the network.
- Prominence should be measured using both direct (adjacent) ties and indirect (or more distant) ties. That is, by paths to other actors with lengths equal or greater than one.
- Knoke and Burt divide this class of measures into two subclasses of prominence:
  - ✓ centrality
  - ✓ prestige

Prominence: Actor Centrality

- Assumptions:
  - ✓ Measurements on a single, symmetric, dichotomous relation.
- If we assume that prominent actors or the most "involved" actors in a network—actively involved in many relationships—then:
  - ✓ We do not differentiate between ties "sent" and ties "received"
  - ✓ We simply are concerned with how involved in network relations each actor is.
- A *central* actor is involved in many nondirectional ties.
- This conceptualization is well suited to measuring things like access and control over resources, and brokerage of information.
- Measures include:
  - ✓ degree centrality
  - ✓ closeness centrality
  - ✓ betweenness centrality
  - ✓ information centrality

Prominence: Degree Centrality

- An actor with high *degree centrality* is “where the action is” in the network since they are in contact or adjacent to many other actors.
- Actors with low degree centrality are more peripheral in the network.
- An isolated actor is so far removed from the network action that removing them from the network has *no* effect on the network and the ties that are present.

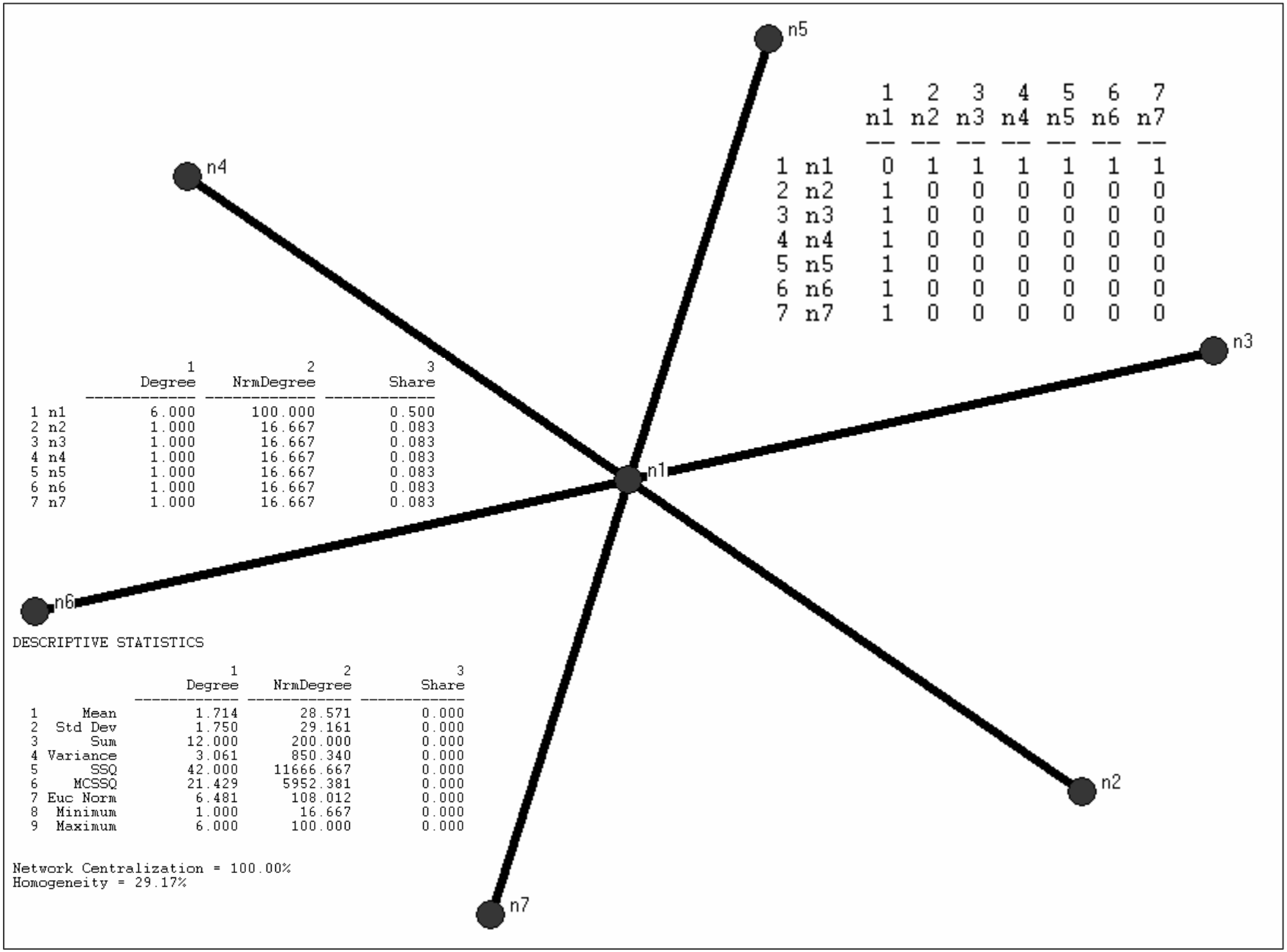
Prominence: Actor Degree Centrality

- The degree of an actor is equal to the number of lines incident to the actor:

$$C_D(n_i) = d(n_i) = x_{i+} = \sum_j x_{ij} = \sum_j x_{ji}$$

- Standardized as a proportion of all possible incident lines:

$$C_D'(n_i) = \frac{d(n_i)}{g-1}$$



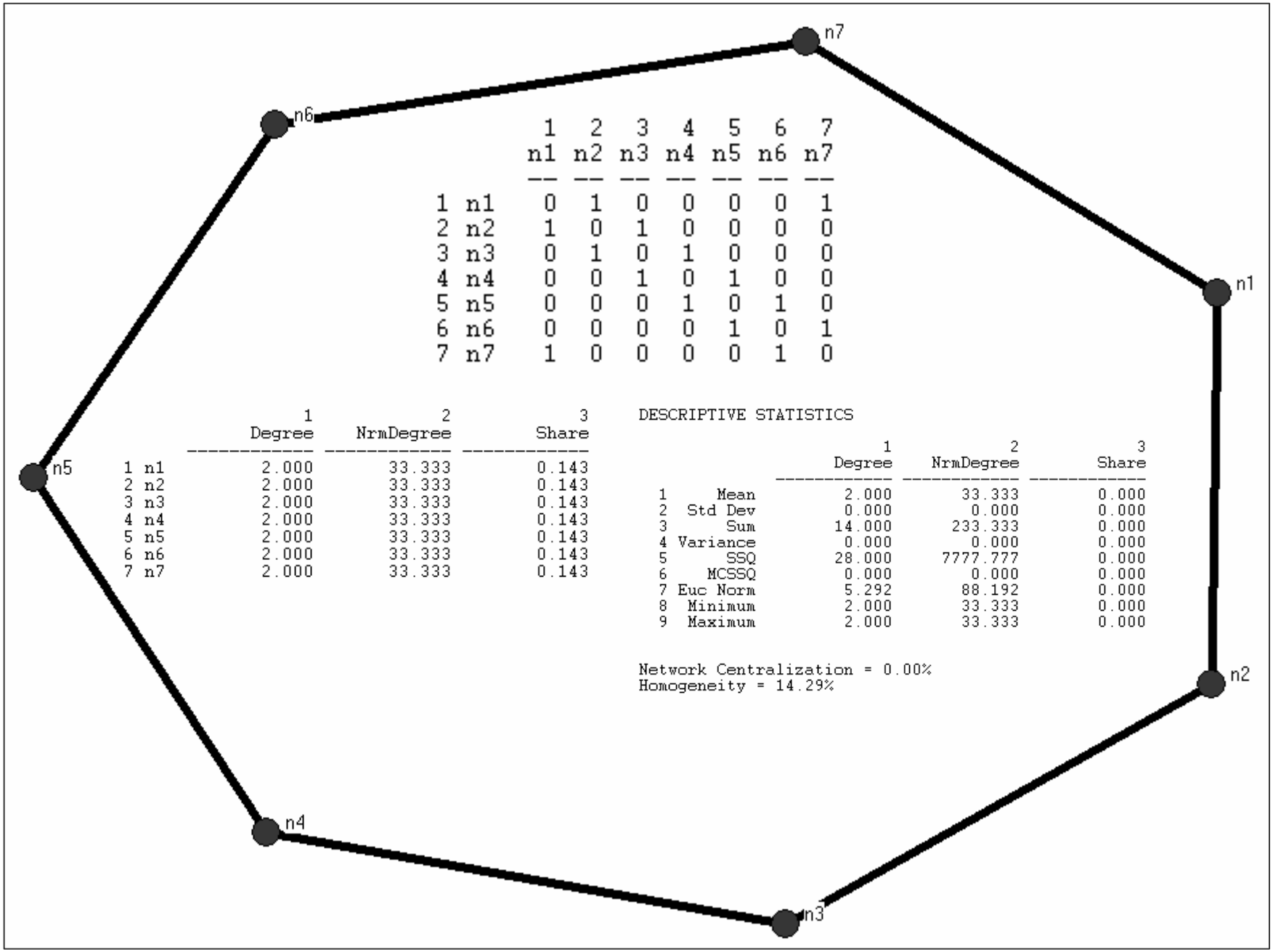
		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	1	1	1	1
2	n2	1	0	0	0	0	0	0
3	n3	1	0	0	0	0	0	0
4	n4	1	0	0	0	0	0	0
5	n5	1	0	0	0	0	0	0
6	n6	1	0	0	0	0	0	0
7	n7	1	0	0	0	0	0	0

		1 Degree	2 NrmDegree	3 Share
1	n1	6.000	100.000	0.500
2	n2	1.000	16.667	0.083
3	n3	1.000	16.667	0.083
4	n4	1.000	16.667	0.083
5	n5	1.000	16.667	0.083
6	n6	1.000	16.667	0.083
7	n7	1.000	16.667	0.083

DESCRIPTIVE STATISTICS

		1 Degree	2 NrmDegree	3 Share
1	Mean	1.714	28.571	0.000
2	Std Dev	1.750	29.161	0.000
3	Sum	12.000	200.000	0.000
4	Variance	3.061	850.340	0.000
5	SSQ	42.000	11666.667	0.000
6	MCSSQ	21.429	5952.381	0.000
7	Euc Norm	6.481	108.012	0.000
8	Minimum	1.000	16.667	0.000
9	Maximum	6.000	100.000	0.000

Network Centralization = 100.00%
Homogeneity = 29.17%



		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	0	0	0	0	1
2	n2	1	0	1	0	0	0	0
3	n3	0	1	0	1	0	0	0
4	n4	0	0	1	0	1	0	0
5	n5	0	0	0	1	0	1	0
6	n6	0	0	0	0	1	0	1
7	n7	1	0	0	0	0	1	0

		1 Degree	2 NrmDegree	3 Share
1	n1	2.000	33.333	0.143
2	n2	2.000	33.333	0.143
3	n3	2.000	33.333	0.143
4	n4	2.000	33.333	0.143
5	n5	2.000	33.333	0.143
6	n6	2.000	33.333	0.143
7	n7	2.000	33.333	0.143

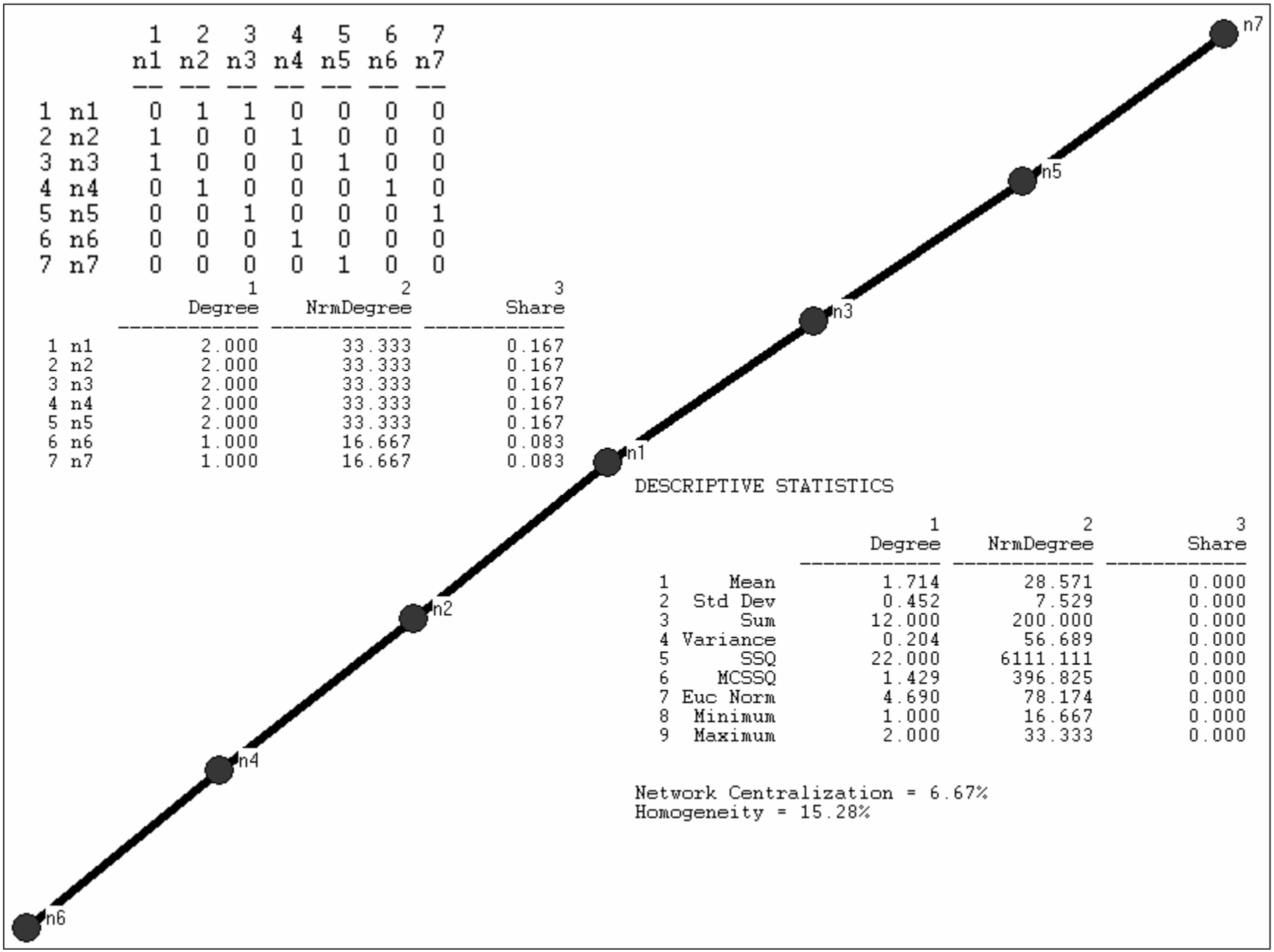
DESCRIPTIVE STATISTICS

		1 Degree	2 NrmDegree	3 Share
1	Mean	2.000	33.333	0.000
2	Std Dev	0.000	0.000	0.000
3	Sum	14.000	233.333	0.000
4	Variance	0.000	0.000	0.000
5	SSQ	28.000	7777.777	0.000
6	MCSSQ	0.000	0.000	0.000
7	Euc Norm	5.292	88.192	0.000
8	Minimum	2.000	33.333	0.000
9	Maximum	2.000	33.333	0.000

Network Centralization = 0.00%
Homogeneity = 14.29%

		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	0	0	0	0
2	n2	1	0	0	1	0	0	0
3	n3	1	0	0	0	1	0	0
4	n4	0	1	0	0	0	1	0
5	n5	0	0	1	0	0	0	1
6	n6	0	0	0	1	0	0	0
7	n7	0	0	0	0	1	0	0

		1 Degree	2 NrmDegree	3 Share
1	n1	2.000	33.333	0.167
2	n2	2.000	33.333	0.167
3	n3	2.000	33.333	0.167
4	n4	2.000	33.333	0.167
5	n5	2.000	33.333	0.167
6	n6	1.000	16.667	0.083
7	n7	1.000	16.667	0.083



DESCRIPTIVE STATISTICS

		1 Degree	2 NrmDegree	3 Share
1	Mean	1.714	28.571	0.000
2	Std Dev	0.452	7.529	0.000
3	Sum	12.000	200.000	0.000
4	Variance	0.204	56.689	0.000
5	SSQ	22.000	6111.111	0.000
6	MCSSQ	1.429	396.825	0.000
7	Euc Norm	4.690	78.174	0.000
8	Minimum	1.000	16.667	0.000
9	Maximum	2.000	33.333	0.000

Network Centralization = 6.67%
Homogeneity = 15.28%

Prominence: Group Degree Centrality

- The degree of a graph or subgraph is equal to:

$$C_D = \frac{\sum_{i=1}^{g}\left[C_D\left(n^*\right) - C_D\left(n_i\right)\right]}{\max\sum_{i=1}^{g}\left[C_D\left(n^*\right) - C_D\left(n_i\right)\right]} = \frac{\sum_{i=1}^{g}\left[C_D\left(n^*\right) - C_D\left(n_i\right)\right]}{(g-1)(g-2)}$$

- This measure has a maximum value of 1 (one actor has ties to all other actors and the other actors only have ties to this actor).
- This measure has a minimum value of 0 (when all actors have equal degrees.

Prominence: Closeness Centrality

- How “close” is an actor to all other actors in the network?
  - ✓ “Close” actors are able to quickly interact with many other actors.
  - ✓ These actors are usually effective in transmitting information.
  - ✓ Closeness is equated to “minimum distance” to other actors—geodesics.
- As geodesics increase in length, centrality decreases. So, the closeness measure is weighted inversely to reflect this.

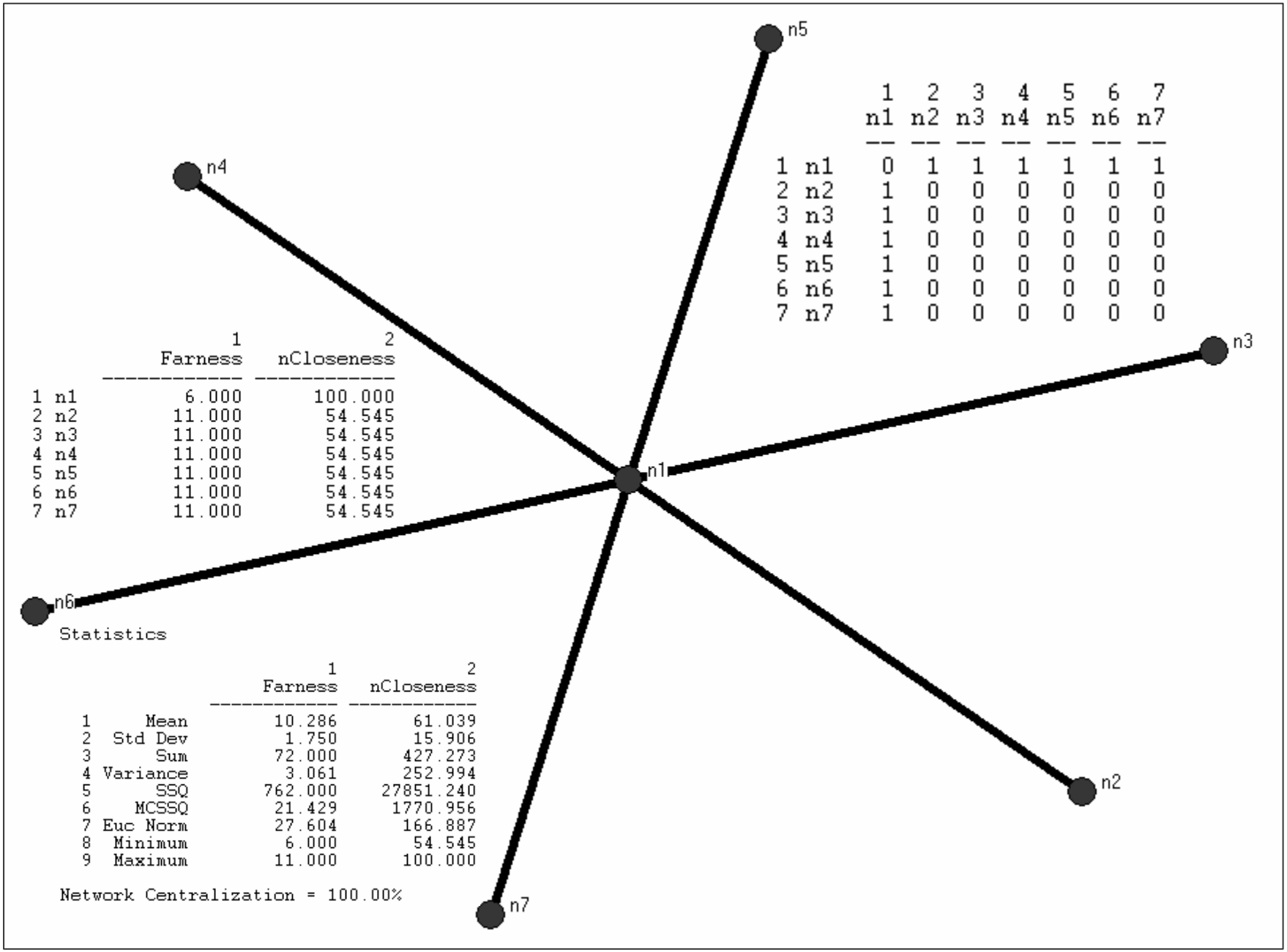
Prominence: Actor Closeness Centrality

- The quantity $d(n_i, n_j)$ is equal to the geodesic length between actors *i* and *j*. The sum of all geodesics from actor *i* to all other *j*'s is called *farness*. C_C, *closeness*, is equal to the inverse of farness.
- The closeness of an actor is equal to:

$$C_C(n_i) = \left[\sum_{j=1}^{g} d(n_i, n_j)\right]^{-1}$$

- Standardized:

$$C'_C(n_i) = \frac{g-1}{\left[\sum_{j=1}^{g} d(n_i, n_j)\right]} = (g-1)C_C(n_i)$$



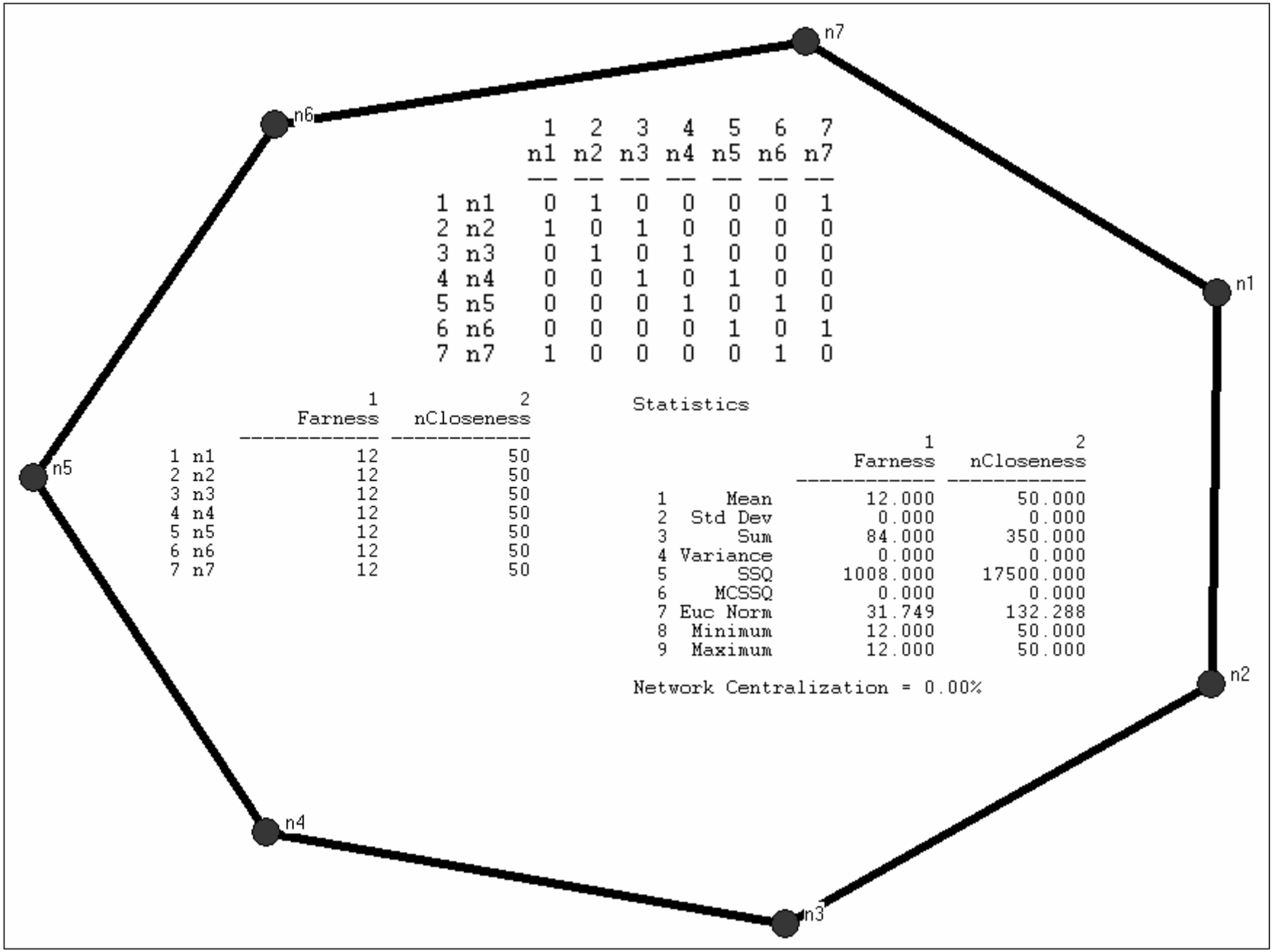
		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	1	1	1	1
2	n2	1	0	0	0	0	0	0
3	n3	1	0	0	0	0	0	0
4	n4	1	0	0	0	0	0	0
5	n5	1	0	0	0	0	0	0
6	n6	1	0	0	0	0	0	0
7	n7	1	0	0	0	0	0	0

		1 Farness	2 nCloseness
1	n1	6.000	100.000
2	n2	11.000	54.545
3	n3	11.000	54.545
4	n4	11.000	54.545
5	n5	11.000	54.545
6	n6	11.000	54.545
7	n7	11.000	54.545

Statistics

		1 Farness	2 nCloseness
1	Mean	10.286	61.039
2	Std Dev	1.750	15.906
3	Sum	72.000	427.273
4	Variance	3.061	252.994
5	SSQ	762.000	27851.240
6	MCSSQ	21.429	1770.956
7	Euc Norm	27.604	166.887
8	Minimum	6.000	54.545
9	Maximum	11.000	100.000

Network Centralization = 100.00%



		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	0	0	0	0	1
2	n2	1	0	1	0	0	0	0
3	n3	0	1	0	1	0	0	0
4	n4	0	0	1	0	1	0	0
5	n5	0	0	0	1	0	1	0
6	n6	0	0	0	0	1	0	1
7	n7	1	0	0	0	0	1	0

		1 Farness	2 nCloseness
1	n1	12	50
2	n2	12	50
3	n3	12	50
4	n4	12	50
5	n5	12	50
6	n6	12	50
7	n7	12	50

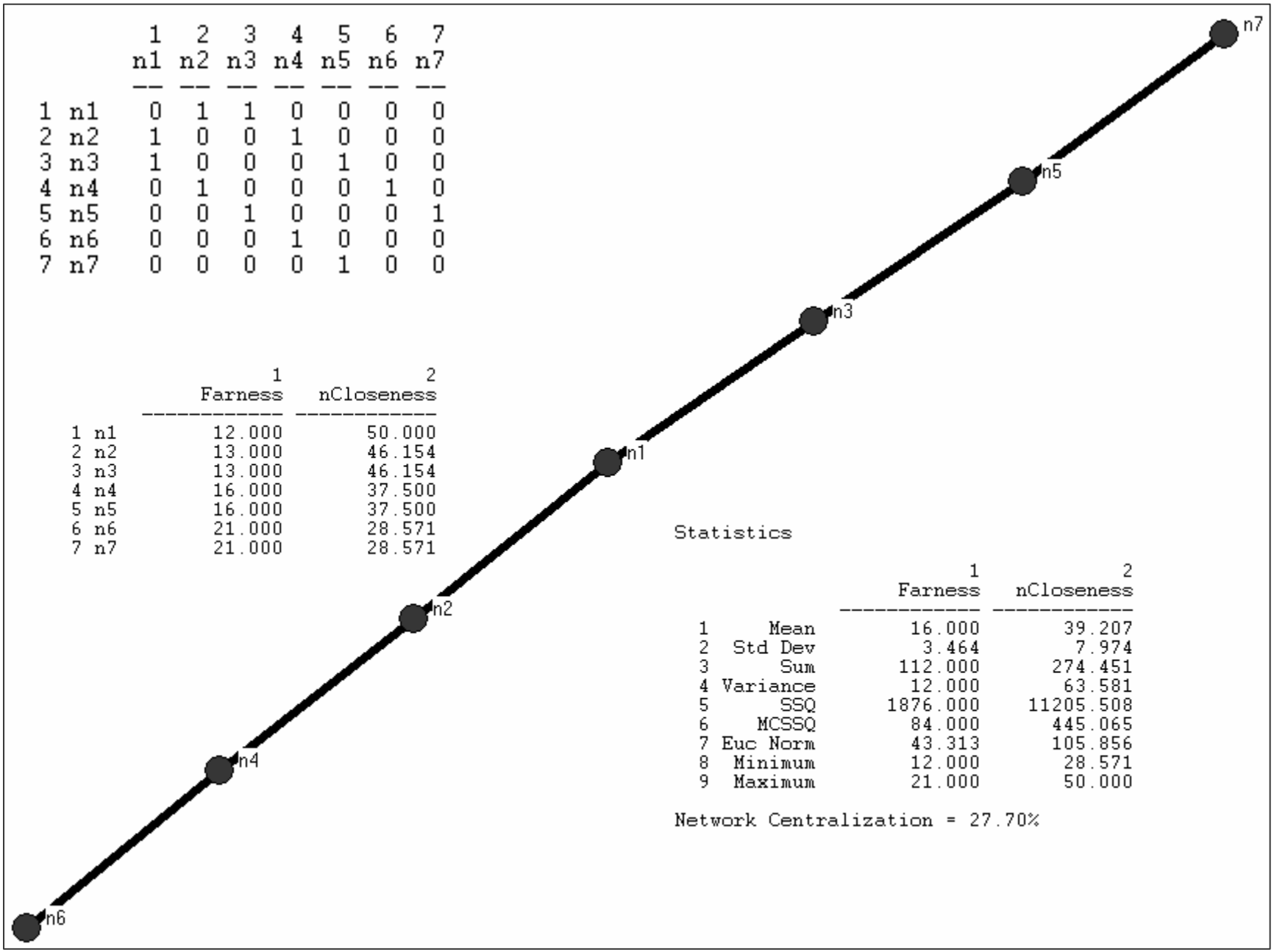
Statistics

		1 Farness	2 nCloseness
1	Mean	12.000	50.000
2	Std Dev	0.000	0.000
3	Sum	84.000	350.000
4	Variance	0.000	0.000
5	SSQ	1008.000	17500.000
6	MCSSQ	0.000	0.000
7	Euc Norm	31.749	132.288
8	Minimum	12.000	50.000
9	Maximum	12.000	50.000

Network Centralization = 0.00%

		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	0	0	0	0
2	n2	1	0	0	1	0	0	0
3	n3	1	0	0	0	1	0	0
4	n4	0	1	0	0	0	1	0
5	n5	0	0	1	0	0	0	1
6	n6	0	0	0	1	0	0	0
7	n7	0	0	0	0	1	0	0

		1 Farness	2 nCloseness
1	n1	12.000	50.000
2	n2	13.000	46.154
3	n3	13.000	46.154
4	n4	16.000	37.500
5	n5	16.000	37.500
6	n6	21.000	28.571
7	n7	21.000	28.571



Statistics

		1 Farness	2 nCloseness
1	Mean	16.000	39.207
2	Std Dev	3.464	7.974
3	Sum	112.000	274.451
4	Variance	12.000	63.581
5	SSQ	1876.000	11205.508
6	MCSSQ	84.000	445.065
7	Euc Norm	43.313	105.856
8	Minimum	12.000	28.571
9	Maximum	21.000	50.000

Network Centralization = 27.70%

Prominence: Group Closeness Centrality

- The closeness of a network is equal to:

$$C_C = \frac{\sum_{i=1}^{g}\left[C'_C\left(n^*\right) - C'_C\left(n_i\right)\right]}{\frac{\left(g-2\right)\left(g-1\right)}{\left(2g-3\right)}}$$

- The numerator is the sum of differences between the maximum standardized actor centrality measure and each actor *i*'s centrality measure.
- The denominator is the theoretical maximum of these summed differences.
- So, this measure ranges from 0 to 1.

Prominence: Betweenness Centrality

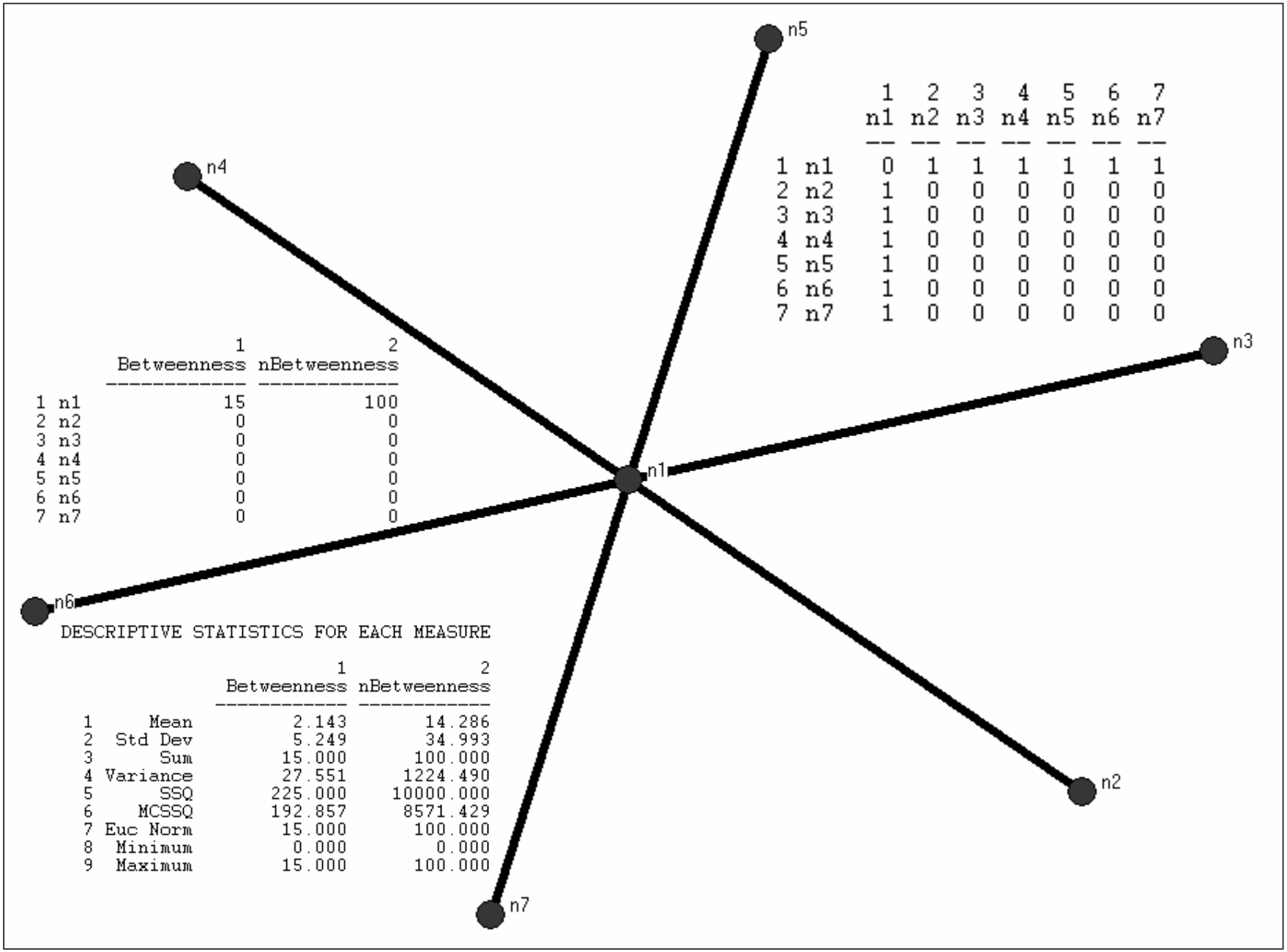
- How "between" is an actor to all other actors in the network?
  - ✓ An actor is central if it lies between other actors on their geodesics.
  - ✓ To have large betweenness centrality, an actor must be between many other actors on their geodesics
- Actors that are between other actors may control interactions between these other actors. That is, they may be *gatekeepers*.
  - ✓ Here, n_2 and n_3 may condition the interaction between n_1 and n_4.

Prominence: Betweenness Centrality

- Betweenness is a measure of the number of times a vertex occurs on a geodesic. The normalized betweenness centrality is the betweenness divided by the maximum possible betweenness expressed as a percentage.
- For any two actors, g_{jk} is number of geodesics linking j and k.
- For three actors, j, k, and i, $g_{jk}(n_i)$ is the number of geodesics between j and k that contain i.
- Betweenness centrality is then summed over all actors:

$$C_B(n_i) = \sum_{j<k} \left(\frac{g_{jk}(n_i)}{g_{jk}} \right)$$

$$C'_B(n_i) = \frac{C_B(n_i)}{\frac{(g-1)(g-2)}{2}}$$

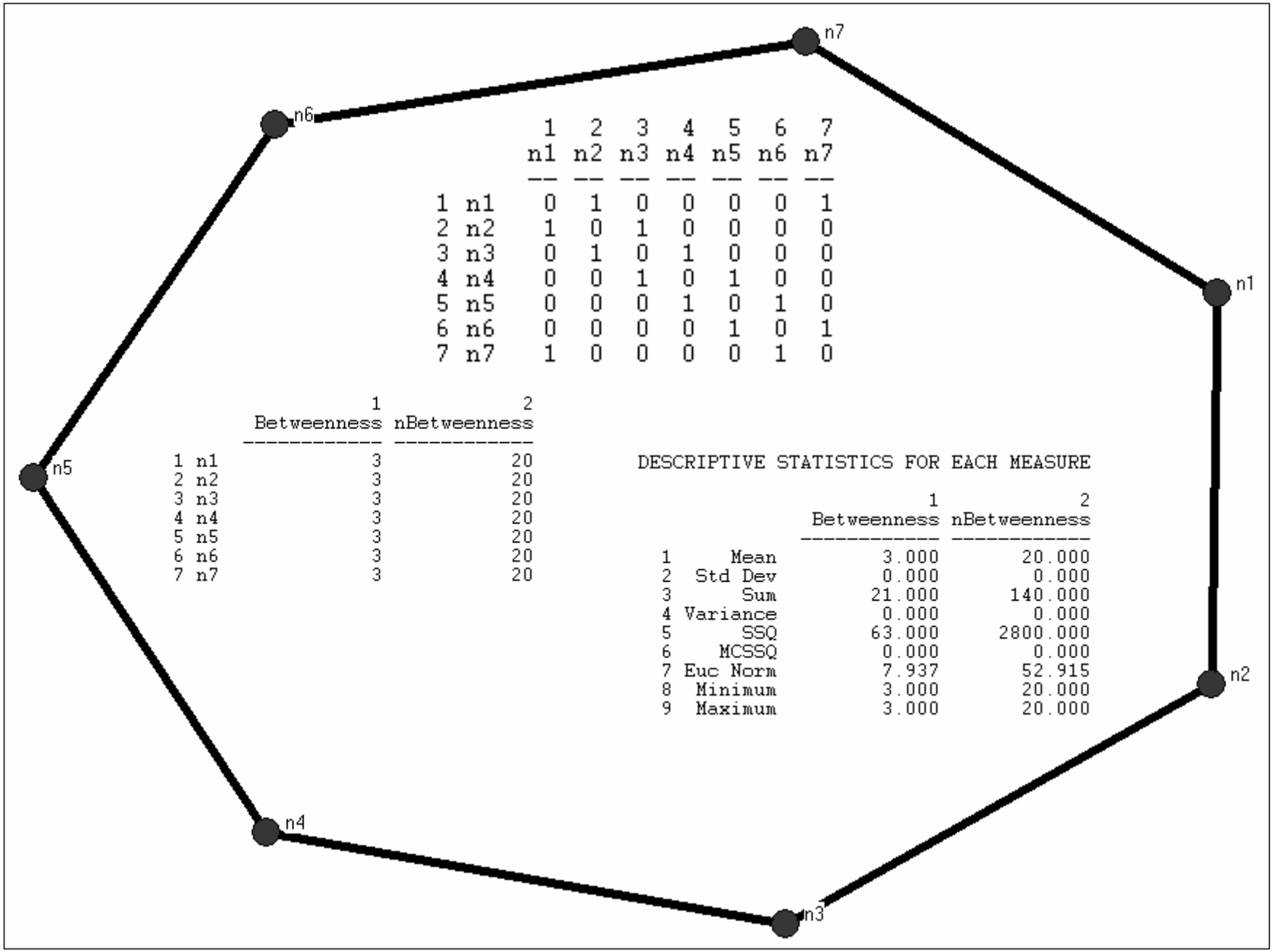


		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	1	1	1	1
2	n2	1	0	0	0	0	0	0
3	n3	1	0	0	0	0	0	0
4	n4	1	0	0	0	0	0	0
5	n5	1	0	0	0	0	0	0
6	n6	1	0	0	0	0	0	0
7	n7	1	0	0	0	0	0	0

		1 Betweenness	2 nBetweenness
1	n1	15	100
2	n2	0	0
3	n3	0	0
4	n4	0	0
5	n5	0	0
6	n6	0	0
7	n7	0	0

DESCRIPTIVE STATISTICS FOR EACH MEASURE

		1 Betweenness	2 nBetweenness
1	Mean	2.143	14.286
2	Std Dev	5.249	34.993
3	Sum	15.000	100.000
4	Variance	27.551	1224.490
5	SSQ	225.000	10000.000
6	MCSSQ	192.857	8571.429
7	Euc Norm	15.000	100.000
8	Minimum	0.000	0.000
9	Maximum	15.000	100.000



		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	0	0	0	0	1
2	n2	1	0	1	0	0	0	0
3	n3	0	1	0	1	0	0	0
4	n4	0	0	1	0	1	0	0
5	n5	0	0	0	1	0	1	0
6	n6	0	0	0	0	1	0	1
7	n7	1	0	0	0	0	1	0

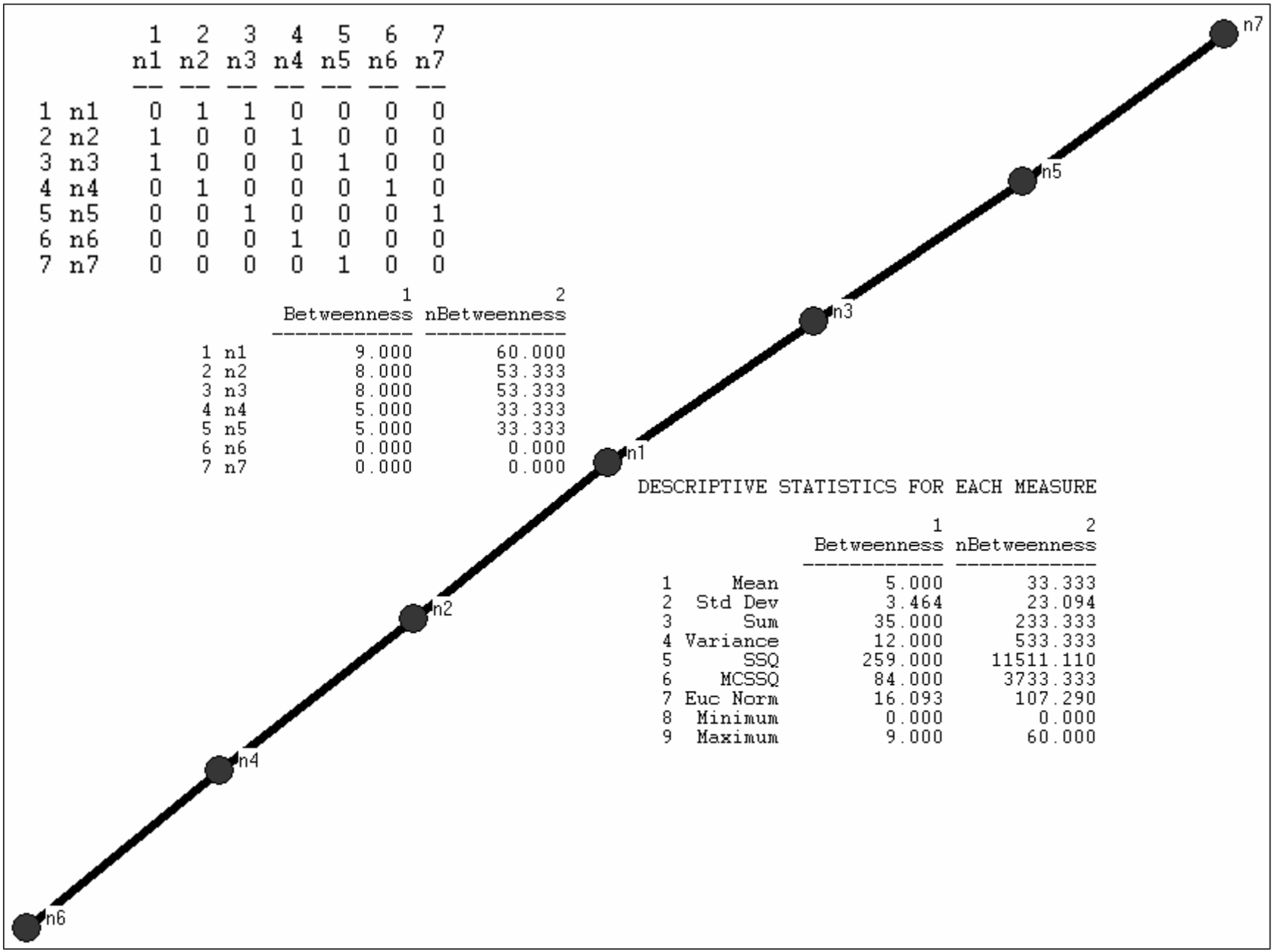
		1 Betweenness	2 nBetweenness
1	n1	3	20
2	n2	3	20
3	n3	3	20
4	n4	3	20
5	n5	3	20
6	n6	3	20
7	n7	3	20

DESCRIPTIVE STATISTICS FOR EACH MEASURE

		1 Betweenness	2 nBetweenness
1	Mean	3.000	20.000
2	Std Dev	0.000	0.000
3	Sum	21.000	140.000
4	Variance	0.000	0.000
5	SSQ	63.000	2800.000
6	MCSSQ	0.000	0.000
7	Euc Norm	7.937	52.915
8	Minimum	3.000	20.000
9	Maximum	3.000	20.000

		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	0	0	0	0
2	n2	1	0	0	1	0	0	0
3	n3	1	0	0	0	1	0	0
4	n4	0	1	0	0	0	1	0
5	n5	0	0	1	0	0	0	1
6	n6	0	0	0	1	0	0	0
7	n7	0	0	0	0	1	0	0

		1 Betweenness	2 nBetweenness
1	n1	9.000	60.000
2	n2	8.000	53.333
3	n3	8.000	53.333
4	n4	5.000	33.333
5	n5	5.000	33.333
6	n6	0.000	0.000
7	n7	0.000	0.000

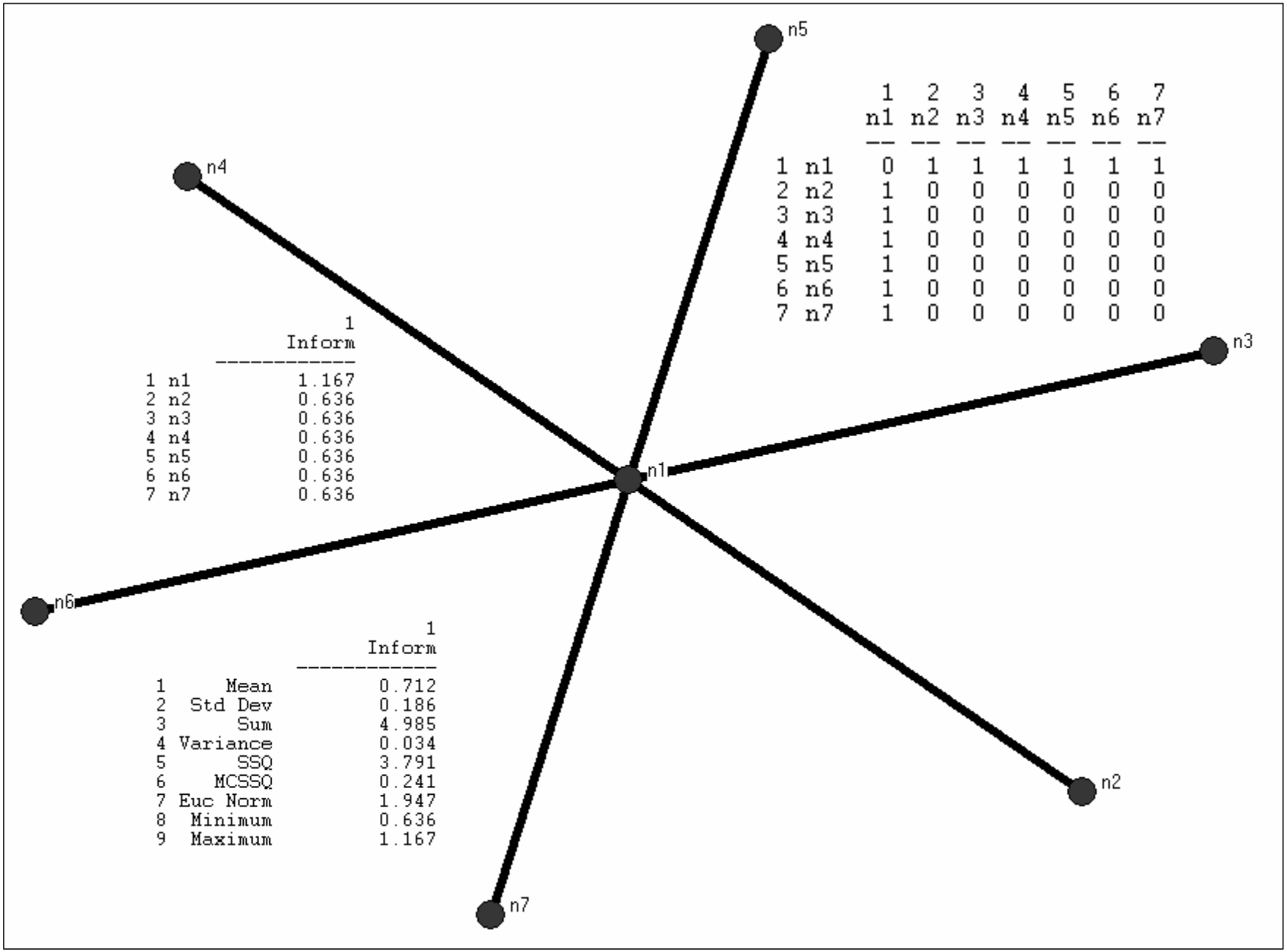


DESCRIPTIVE STATISTICS FOR EACH MEASURE

		1 Betweenness	2 nBetweenness
1	Mean	5.000	33.333
2	Std Dev	3.464	23.094
3	Sum	35.000	233.333
4	Variance	12.000	533.333
5	SSQ	259.000	11511.110
6	MCSSQ	84.000	3733.333
7	Euc Norm	16.093	107.290
8	Minimum	0.000	0.000
9	Maximum	9.000	60.000

Prominence: Information Centrality

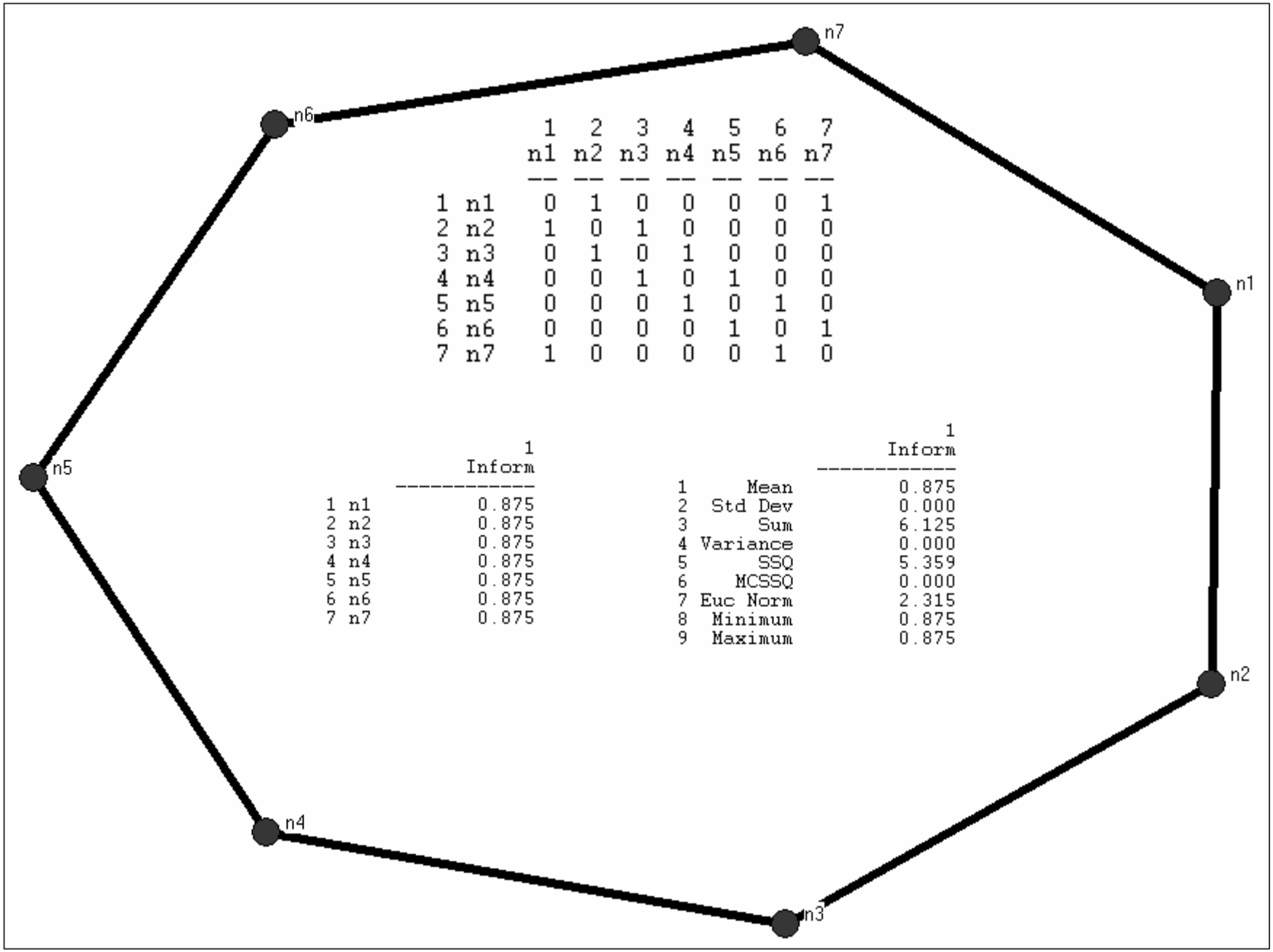
- Betweenness centrality asks: “How “between” is an actor to all other actors in the network?
  - ✓ An actor is central if it lies between other actors on their geodesics.
  - ✓ To have large betweenness centrality, an actor must be between many other actors on their geodesics
- This explicitly weights each geodesic path as equal to each other and ignores other paths.
- *Information centrality* takes into account:
  - ✓ Geodesics weighted as unity, and
  - ✓ Longer paths are weighted less than the geodesics (the inverse of their length)
  - ✓ This measure assess *all* paths between two points and weights them by their length.



		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	1	1	1	1
2	n2	1	0	0	0	0	0	0
3	n3	1	0	0	0	0	0	0
4	n4	1	0	0	0	0	0	0
5	n5	1	0	0	0	0	0	0
6	n6	1	0	0	0	0	0	0
7	n7	1	0	0	0	0	0	0

		1 Inform
1	n1	1.167
2	n2	0.636
3	n3	0.636
4	n4	0.636
5	n5	0.636
6	n6	0.636
7	n7	0.636

		1 Inform
1	Mean	0.712
2	Std Dev	0.186
3	Sum	4.985
4	Variance	0.034
5	SSQ	3.791
6	MCSSQ	0.241
7	Euc Norm	1.947
8	Minimum	0.636
9	Maximum	1.167



		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	0	0	0	0	1
2	n2	1	0	1	0	0	0	0
3	n3	0	1	0	1	0	0	0
4	n4	0	0	1	0	1	0	0
5	n5	0	0	0	1	0	1	0
6	n6	0	0	0	0	1	0	1
7	n7	1	0	0	0	0	1	0

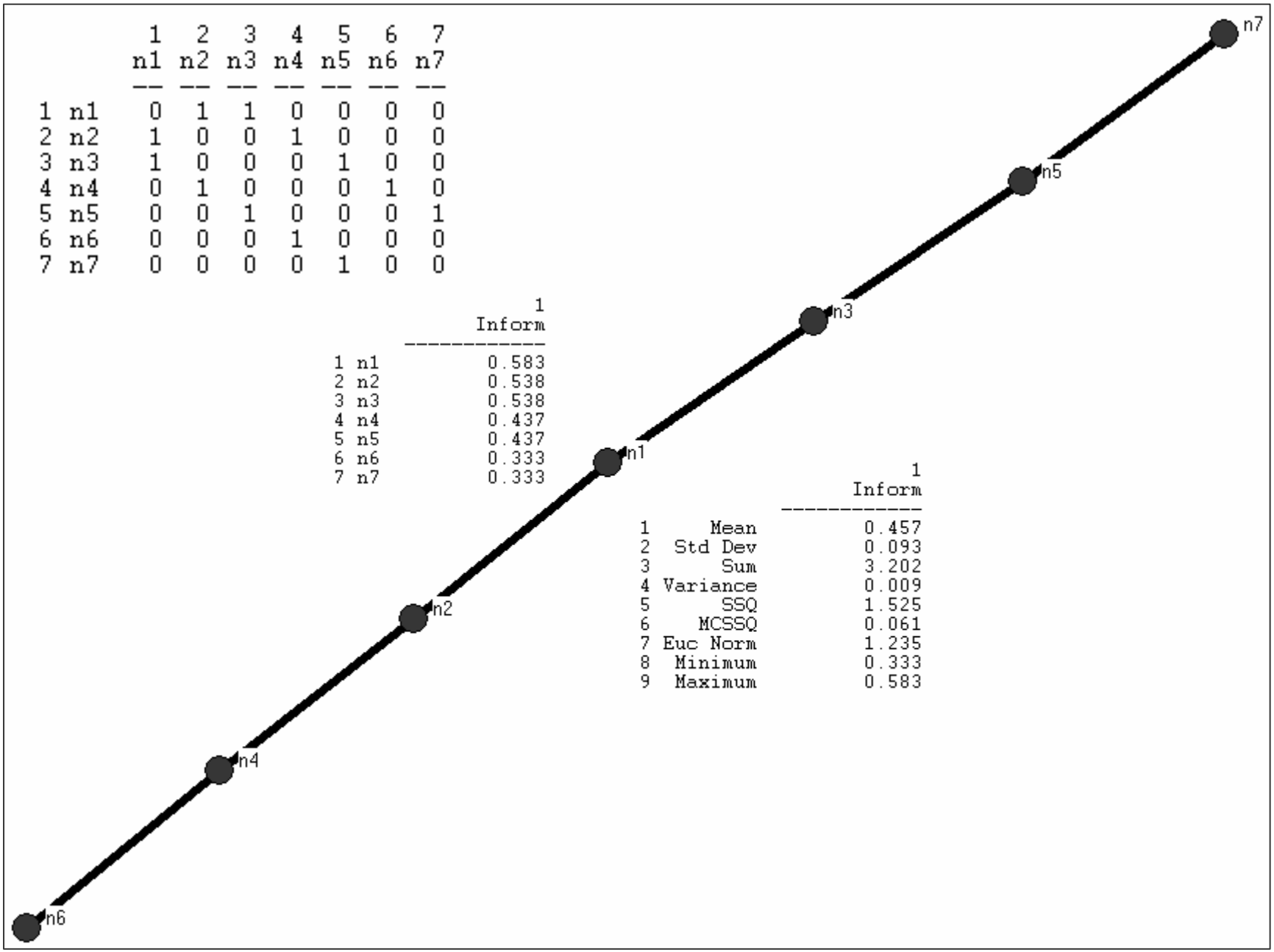
		1 Inform
1	n1	0.875
2	n2	0.875
3	n3	0.875
4	n4	0.875
5	n5	0.875
6	n6	0.875
7	n7	0.875

		1 Inform
1	Mean	0.875
2	Std Dev	0.000
3	Sum	6.125
4	Variance	0.000
5	SSQ	5.359
6	MCSSQ	0.000
7	Euc Norm	2.315
8	Minimum	0.875
9	Maximum	0.875

		1 n1	2 n2	3 n3	4 n4	5 n5	6 n6	7 n7
1	n1	0	1	1	0	0	0	0
2	n2	1	0	0	1	0	0	0
3	n3	1	0	0	0	1	0	0
4	n4	0	1	0	0	0	1	0
5	n5	0	0	1	0	0	0	1
6	n6	0	0	0	1	0	0	0
7	n7	0	0	0	0	1	0	0

		1 Inform
1	n1	0.583
2	n2	0.538
3	n3	0.538
4	n4	0.437
5	n5	0.437
6	n6	0.333
7	n7	0.333

		1 Inform
1	Mean	0.457
2	Std Dev	0.093
3	Sum	3.202
4	Variance	0.009
5	SSQ	1.525
6	MCSSQ	0.061
7	Euc Norm	1.235
8	Minimum	0.333
9	Maximum	0.583



Prominence: Actor Prestige

- Assumptions:
  - ✓ Measurements on a single, asymmetric, dichotomous relation.
- What if we distinguish between ties sent and ties received?
- A *prestigious* actor receives many ties from other actors.
- Actor prestige increases as the actor becomes the object of more ties.
- Measures include:
  - ✓ centrality
    - » degree
    - » closeness
  - ✓ prestige
    - » degree prestige
    - » proximity prestige
    - » status or rank prestige