EIGENVECTOR CENTRALITY

Eigenvector centrality is a measure of the importance of a node in a network. It assigns relative scores to all nodes in the network based on the principle that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. Google's PageRank is a variant of the Eigenvector centrality measure.

Using the adjacency matrix to find eigenvector centrality

Let x_i denote the score of the i^{th} node. Let $A = (a_{i,j})$ be the adjacency matrix of the network. Hence $a_{i,j} = 1$ if the i^{th} node is linked to the j^{th} node, and $a_{i,j} = 0$ otherwise. More generally, the entries in A can be real numbers representing connection strengths, as in a stochastic matrix.

For the i^{th} node, let the centrality score be proportional to the sum of the scores of all nodes which are connected to it. Hence

$$x_i = \frac{1}{\lambda} \sum_{j \in M(i)} x_j = \frac{1}{\lambda} \sum_{j=1}^{N} a_{i,j} x_j$$

where $M(i)$ is the set of nodes that are connected to the i^{th} node, N is the total number of nodes and λ is a constant. In vector notation this can be rewritten as

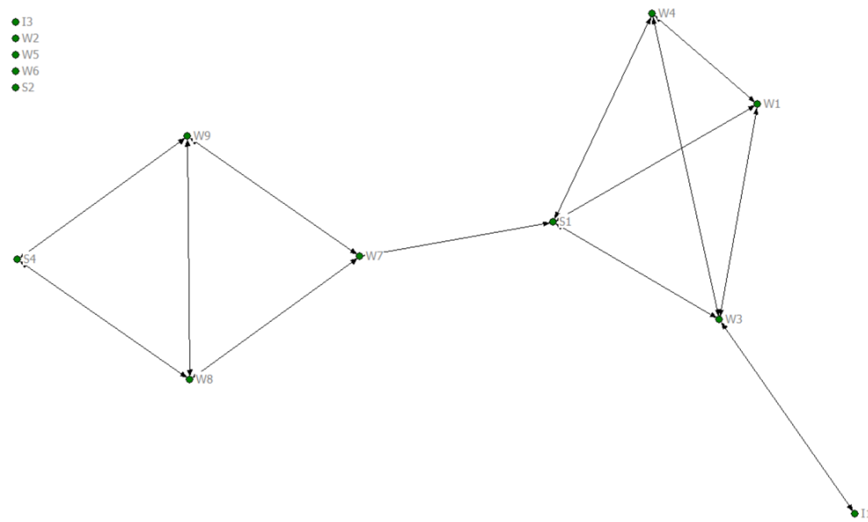
$$\mathbf{x} = \frac{1}{\lambda} \mathbf{A} \mathbf{x}$$

or as the eigenvector equation $\mathbf{A}\mathbf{x} = \lambda \mathbf{x}$

In general, there will be many different eigenvalues λ for which an eigenvector solution exists. However, the additional requirement that all the entries in the eigenvector be positive implies (by the Perron–Frobenius theorem) that only the greatest eigenvalue results in the desired centrality measure. The i^{th} component of the related eigenvector then gives the centrality score of the i^{th} node in the network.

(wikipedia.org/wiki/centrality)

Wiring Room



$$C_{S4} = \frac{1}{\lambda} (C_{W9} + C_{W8})$$

$$C_{W9} = \frac{1}{\lambda} (C_{S4} + C_{W8} + C_{W7})$$

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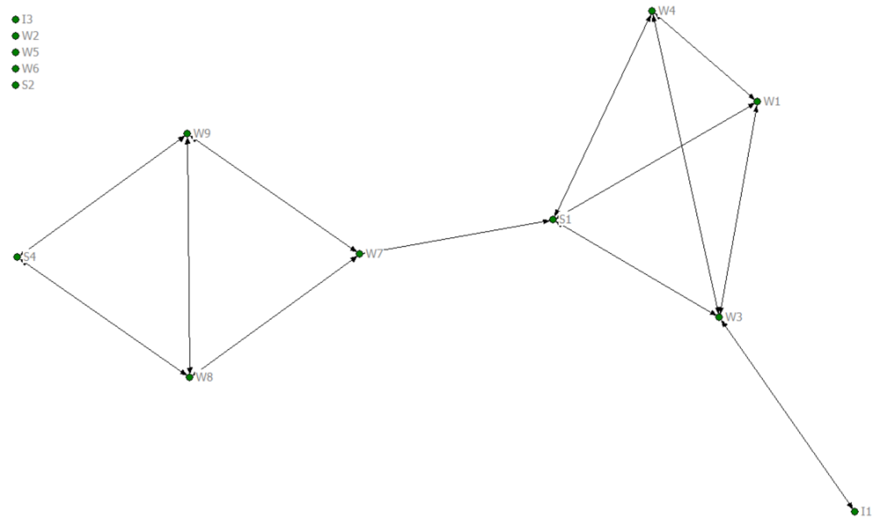
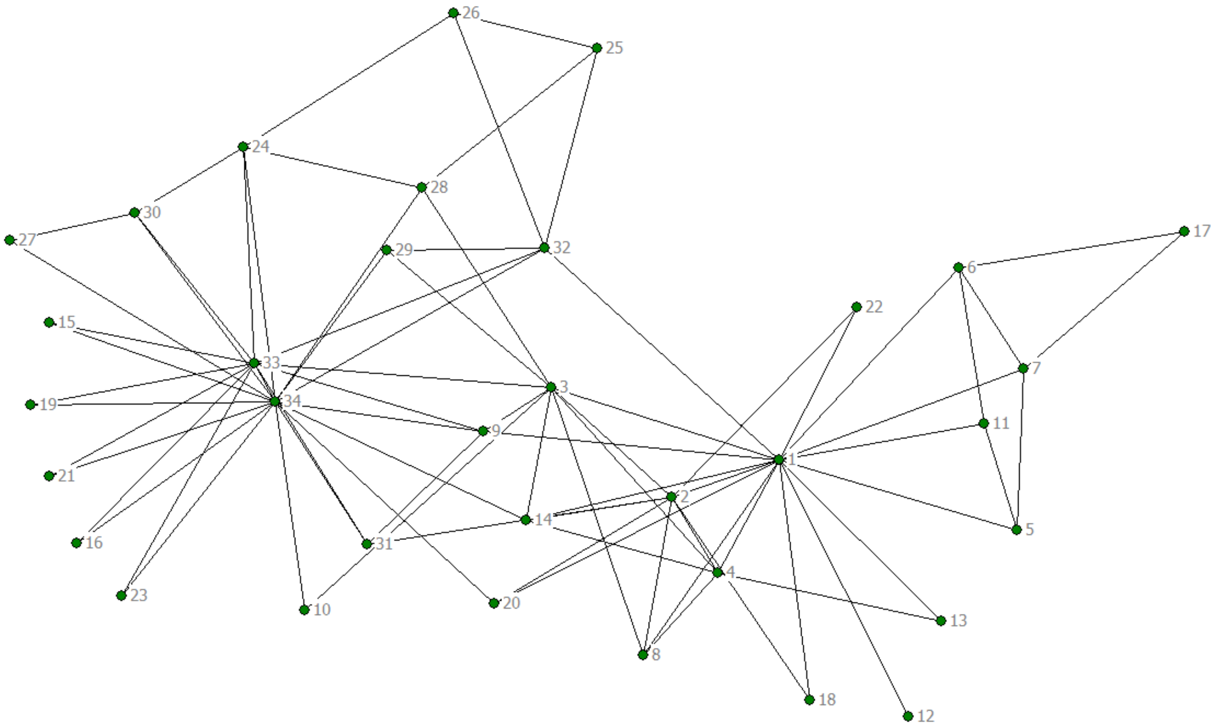
$$C_{I1} = \frac{1}{\lambda} (C_{W3})$$

Wiring Room



Eigenvector Centrality	Degree Centrality	Closeness Centrality	Betweenness Centrality
S1 (.498) W3 (.472) W1,W4 (.438) W7 (.254) W8,W9 (.159) I1 (.147) S4 (.099)	W3,S1 W9,W8,W7,W1,W4 S4 I1	S1 W7 W3 W1,W4 W8, W9 I1 S4	S1 W7 W3 W8,W9 W1,I1,W4,S4 (0)

Karate Club



Eigenvector Centrality	Degree Centrality	Closeness Centrality	Betweenness Centrality
1 (.355)	34 (17)	1	1 (231)
3 (.317)	1 (16)	33	34 (161)
2 (.266)	33 (12)	34	33 (77)
9 (.227)	3 (10)	329,14,33	3 (76)
14 (.226)	2 (9)	20	32 (73)
4 (.211)	4,32 (6)	2	9 (30)
31 (.175)	9,14,24	4	.
8 (.171)	6,7,8,28,30,31	.	.
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Martin Everett Problem

Noah Friedkin

From: M.Everett@gre.ac.uk [M.G.Everett@greenwich.ac.uk]
Sent: Tuesday, June 08, 1999 5:17 AM
To: SOCNET@lists.ufl.edu
Subject: Terminal node

In the last issue of connections Steve Borgatti gave us a 19 actor network in which the most central actors in terms of degree, closeness, betweenness and eigenvector were all different. It would be interesting to know what the smallest example of such a network is. I have found the following 13 actor network which has the same property. I'm sure some-one can find a smaller one.....these are very useful in teaching students the difference between the various centrality measures.

Adjacency matrix of a 13 node network which has the four distinct aspects of centrality.

```
0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0
1 1 1 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 1 1 0 1 0 0 0 0 0 0
0 0 0 0 0 1 0 1 1 0 0 0 0
0 0 0 0 0 0 1 0 1 1 0 0 0
0 0 0 0 0 0 1 1 0 0 1 0 0
0 0 0 0 0 0 0 1 0 0 1 1 0
0 0 0 0 0 0 0 0 1 1 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 1
0 0 0 0 0 0 0 0 0 0 0 1 0
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Note the actor with the highest degree is 4, highest betweenness is 6, highest closeness is 7 and highest eigenvector is 8.

I throw down the challange......can we find a smaller example!!

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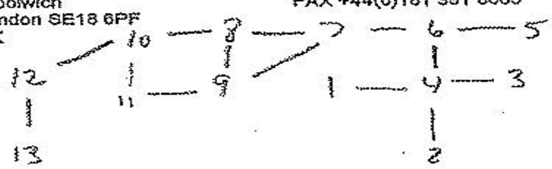


Eigenvector Centrality	Degree Centrality	Closeness Centrality	Betweenness Centrality
8 (.470) 9 (.458) 7 (.451) 10 (.346) 11 (.301) 6 (.274) 4 (.177) 12 (.151) 5 (.103) 1,2,3 (.067) 13 (.056)	4 6,7,8,9,10 11,12 1,2,3,5,13	7 6 8 9 4 10 11 5 1,2,3,12 13	6 7 4 8 10 12 9 11 1,2,3,5,13 (0)

Medieval Russian Trade Network

Figure 1. *Russian trade routes in the 12th - 13th centuries.*

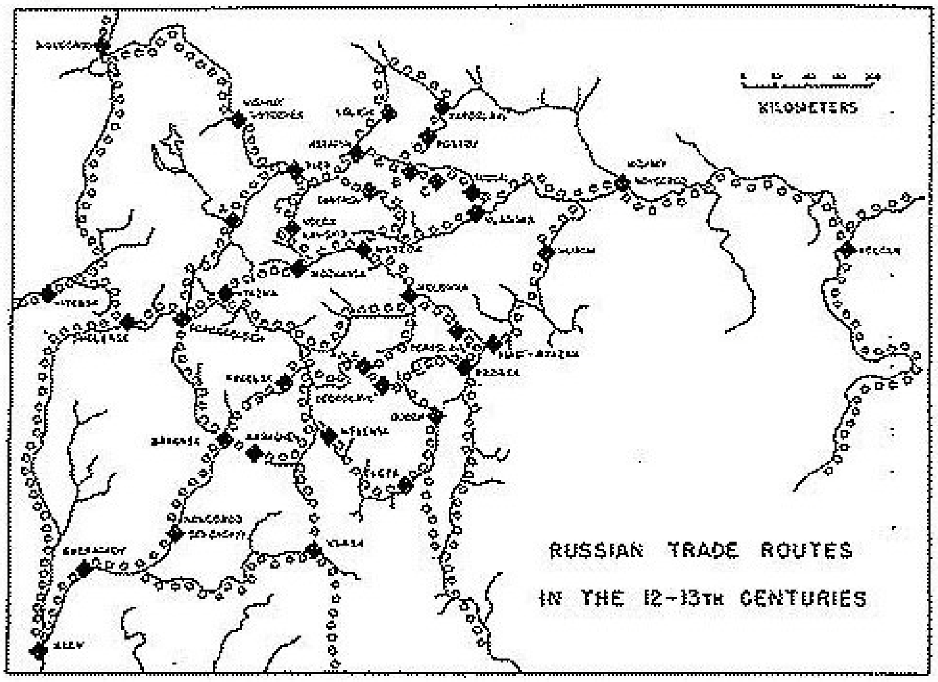


Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*

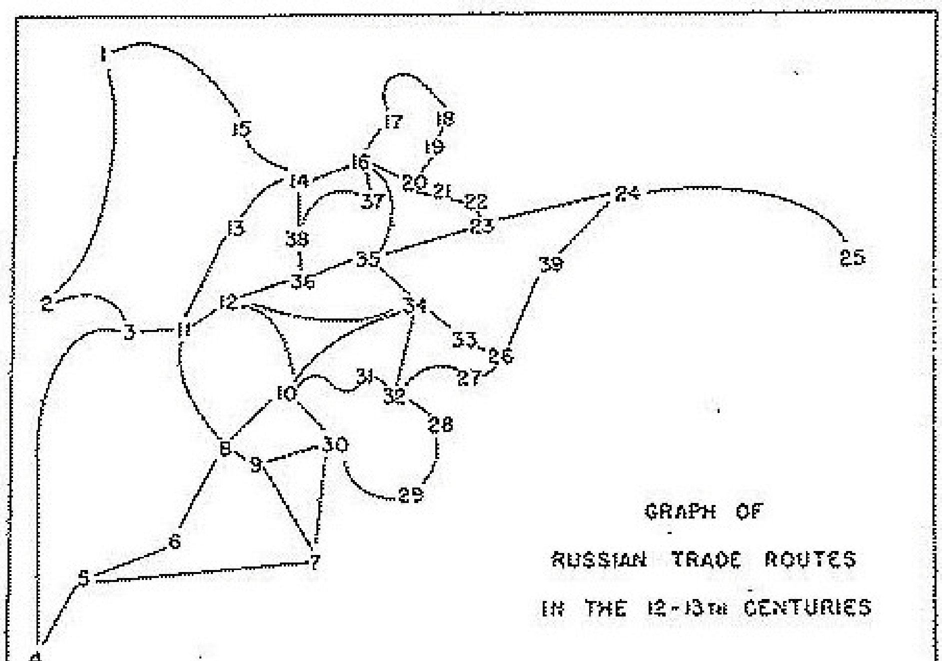
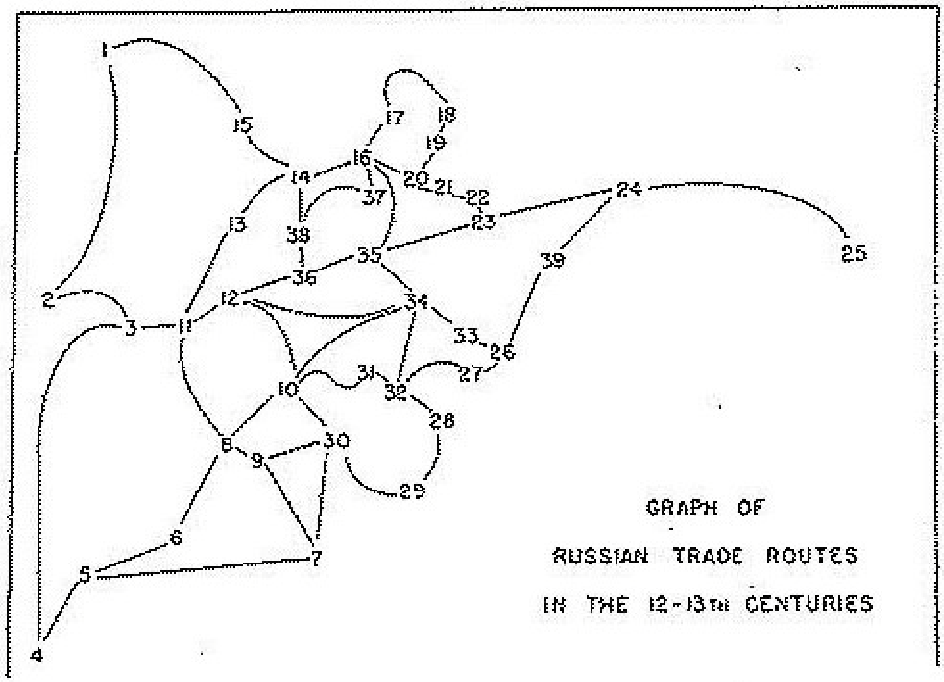


Figure 2. *Graph of Russian trade routes in the 12th - 13th centuries.*



Eigenvector Centrality	Degree Centrality	Closeness Centrality	Betweenness Centrality
10 (.405)	10,16,34 (5)	34	34
34 (.381)	8,11,12,14,30,32,35	35	35
12 (.350)	.	12	16
8 (.265)	.	10	10
30 (.239)	.	16,36	11
35 (.237)		11	14
11 (.231)		14	23
32 (.208)		.	12
36 (.201)		.	.
.		.	.
.			.
.			

35 is Moscow
34 is Kolomna

See http://www.analytictech.com/networks/pitts.htm on the controversy.